Rock pulverization at extreme strain rate near the San Andreas Fault

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Supplementary material

Strain rate near the tip of a rupture front

In this supplementary material, we discuss the maximum strain rate that can be obtained near a mode II rupture front. As we want to get the maximum strain rate, we consider a pure elastic case, not taking into account the viscoelastic behaviour inside the process zone at the fracture tip.

The stress at a distance y_0 from the fault core is computed in a 2D model within a referential frame moving with the rupture tip. There is a directivity effect that can be interpreted as a "relativistic contraction", where the following parameters appear:

$$\alpha_d = \sqrt{1-\frac{v^2}{c_d^2}} \quad \alpha_s = \sqrt{1-\frac{v^2}{c_s^2}} .$$

c_d is the velocity of dilatational waves along the (x,y) plan and c_s is the velocity of shear wave within this plan. Independently of the choice of the boundary conditions, we have

$$c_s = \sqrt{\mu/\rho}$$

where μ is the shear modulus, and ρ is the density. On the other hand, c_d depends on the boundary conditions. For plane stress, we get

$$c_d = \sqrt{\frac{2}{1-\nu}}\, c_s,$$

(ν is the Poisson ratio) while for plane strain, we get

$$c_d = \sqrt{\frac{2(1-\nu)}{1-2\nu}}\, c_s.$$

with the coordinates of this moving frame (η=x-v×t, ζ=y0), one can define reduced coordinates for both dilatational and shear waves. In polar coordinates ($r_d = \gamma_d\, r,\ \theta_d$):

$$\gamma_d\, r e^{i\theta_d} = \eta + i\alpha_d \zeta$$
$$\gamma_s\, r e^{i\theta_s} = \eta + i\alpha_s \zeta$$

As we are interested in distance (~100m) small relative to the rupture size (several 10s of kilometres for large earthquakes), we will use a Taylor expansion in power of *r*, where *r* is the distance to the fracture.

Freund gives the first order, which is singular in $1/r^{½}$:

$$\sigma_{xx} = \frac{K_{II}}{\sqrt{2\pi r}D}\left[\left(1+2\alpha_d^2-\alpha_s^2\right)\frac{\sin\left(\frac{1}{2}\theta_d\right)}{\sqrt{\gamma_d}} - \left(1+\alpha_s^2\right)\frac{\sin\left(\frac{1}{2}\theta_s\right)}{\sqrt{\gamma_s}}\right]$$
$$\sigma_{yy} = \frac{K_{II}}{\sqrt{2\pi r}D}\left[2\alpha_s\left(1+\alpha_s^2\right)\frac{\sin\left(\frac{1}{2}\theta_d\right)}{\sqrt{\gamma_d}} - 2\alpha_s\left(1+\alpha_s^2\right)\frac{\sin\left(\frac{1}{2}\theta_s\right)}{\sqrt{\gamma_s}}\right]$$
$$\sigma_{xy} = \frac{K_{II}}{\sqrt{2\pi r}D}\left[4\alpha_d\alpha_s\frac{\cos\left(\frac{1}{2}\theta_d\right)}{\sqrt{\gamma_d}} - \left(1+\alpha_s^2\right)\frac{\cos\left(\frac{1}{2}\theta_s\right)}{\sqrt{\gamma_s}}\right]$$

This expression only depends on the instantaneous value of the stress intensity factor K_{II}, not on its time derivatives. The current rupture front velocity intervenes within several parameters: α_d, α_s *r* and $D = 4\alpha_d\alpha_s - \left(1+\alpha_s^2\right)^2$. These parameters are defined while *D>0* and $0<v<c_s<c_d$, or equivalently $v<c_R$, where c_R is the Rayleigh velocity, which is the non-null rupture velocity that satisfies *D=0*.

Supposing plane strain conditions ($\varepsilon_{zz} = 0$)

$$\varepsilon_{xx} = \frac{1-\nu^2}{E}\sigma_{xx} - \frac{\nu(1+\nu)}{E}\sigma_{yy}$$
$$\varepsilon_{yy} = -\frac{\nu(1+\nu)}{E}\sigma_{xx} + \frac{1-\nu^2}{E}\sigma_{yy}$$
$$\varepsilon_{xy} = \frac{1}{2\mu}\sigma_{xy}$$

Supposing plane stress condition ($\sigma_{zz} = 0$)

$$\varepsilon_{xx} = \frac{1}{E}\left(\sigma_{xx} - \nu\sigma_{yy}\right)$$
$$\varepsilon_{yy} = \frac{1}{E}\left(\sigma_{yy} - \nu\sigma_{xx}\right)$$
$$\varepsilon_{xy} = \frac{1}{2\mu}\sigma_{xy}$$

We get therefore the following expression for the volumetric strain $\Delta=\varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz}$:

$$\Delta = \frac{(1+\nu)(1-2\nu)}{E}\frac{2K_{II}}{\sqrt{2\pi r}D}\alpha_s\left(\alpha_s^2-\alpha_d^2\right)\frac{\sin{\theta_d}/{2}}{\sqrt{\gamma_d}} \quad \cdots\cdots \text{ (plane strain)}$$
$$\Delta = \frac{1-2\nu}{E}\frac{2K_{II}}{\sqrt{2\pi r}D}\alpha_s\left(\alpha_s^2-\alpha_d^2\right)\frac{\sin{\theta_d}/{2}}{\sqrt{\gamma_d}} \quad \cdots\cdots \text{ (plane stress)}$$

We can then deduce the time derivative of volumetric strain. Supposing that the rupture tip is propagating at a constant speed *v*, the time dependence is through the spatial terms r, γ_i and θ_i. We get

$$\frac{d\Delta}{dt} = \frac{(1+\nu)(1-2\nu)}{E}\frac{2K_{II}}{\sqrt{2\pi}D}\alpha_s\left(\alpha_s^2-\alpha_d^2\right)v\frac{\sin({3\theta_d}/{2})}{(\gamma_d r)^{5/2}} \quad \cdots\cdots \text{ (plane strain)}$$
$$\frac{d\Delta}{dt} = \frac{1-2\nu}{E}\frac{2K_{II}}{\sqrt{2\pi}D}\alpha_s\left(\alpha_s^2-\alpha_d^2\right)v\frac{\sin({3\theta_d}/{2})}{(\gamma_d r)^{5/2}} \quad \cdots\cdots \text{ (plane stress)}$$

On the four above equations, there is the same constant $\frac{k}{E}\frac{2K_{II}}{\sqrt{2\pi}D}\alpha_s\left(\alpha_s^2-\alpha_d^2\right)v$, with $k=(1+\nu)(1-2\nu)$ or $k = 1-2\nu$. The time dependence is only present through the location terms $r, \gamma_d, \theta, \theta_d$. The constant involve the stress intensity factor K_{II} that is poorly known. On the other hand, the conservation of the initial structure of the granodiorite requires they were submitted to a small strain. Therefore, we chose to normalize both Δ and $\partial\Delta/\partial t$ so that the maximum strain is 2%, to assess what maximum strain rate is reachable with this small strain condition. We plot $\partial\Delta/\partial t$ versus Δ in figure 4 of the main text.

We get also the maximum shear strain in the (x,y) plane as $\varepsilon_s = \sqrt{\left(\frac{\varepsilon_{xx} - \varepsilon_{yy}}{2}\right)^2 + \varepsilon_{xy}{}^2}$.

$$\varepsilon_s = \frac{1}{2\mu}\frac{K_{II}}{\sqrt{2\pi}D}\sqrt{\left(4\alpha_d\alpha_s\frac{\cos\left(\frac{\theta_d}{2}\right)}{\sqrt{\gamma_d r}} - \left(1+\alpha_s^2\right)\frac{\cos\left(\frac{\theta_s}{2}\right)}{\sqrt{\gamma_s r}}\right)^2 + 4\alpha_s^2\left(\left(1+\alpha_d^2\right)\frac{\sin\left(\frac{\theta_d}{2}\right)}{\sqrt{\gamma_d r}} - \left(1+\alpha_s^2\right)\frac{\sin\left(\frac{\theta_s}{2}\right)}{\sqrt{\gamma_s r}}\right)^2} \quad \cdots\cdots \text{(plane strain)}$$

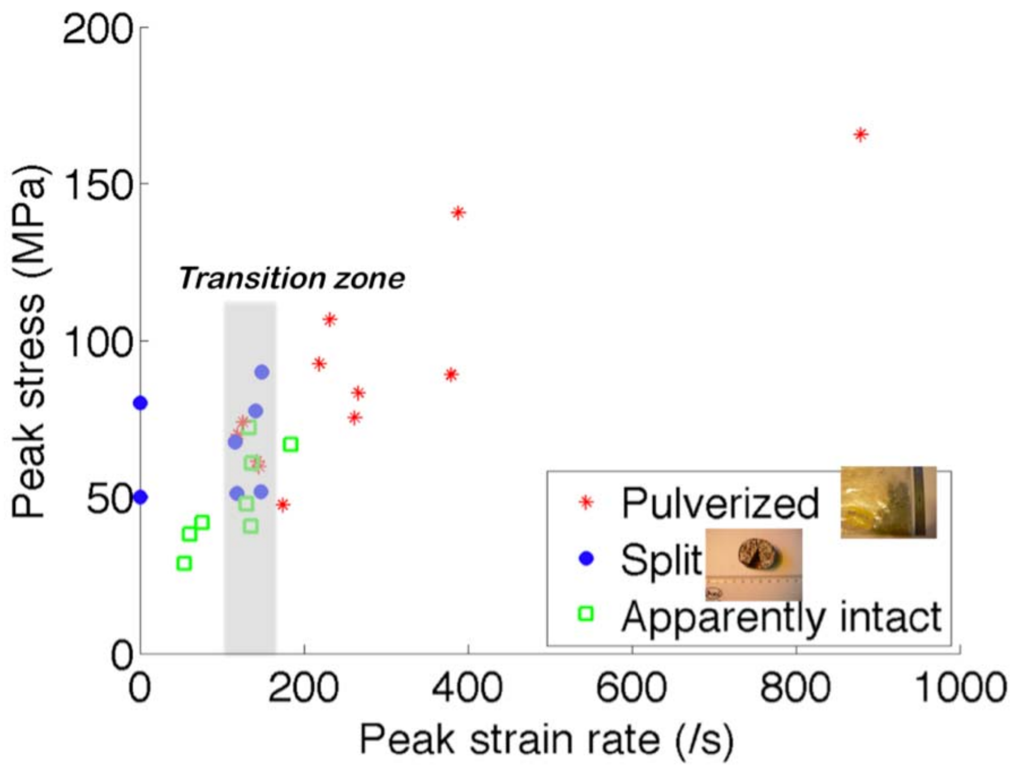
Its time derivative can be computed analytically, but this large expression is cumbersome, so that we computed it with finite differences.

Higher order terms.

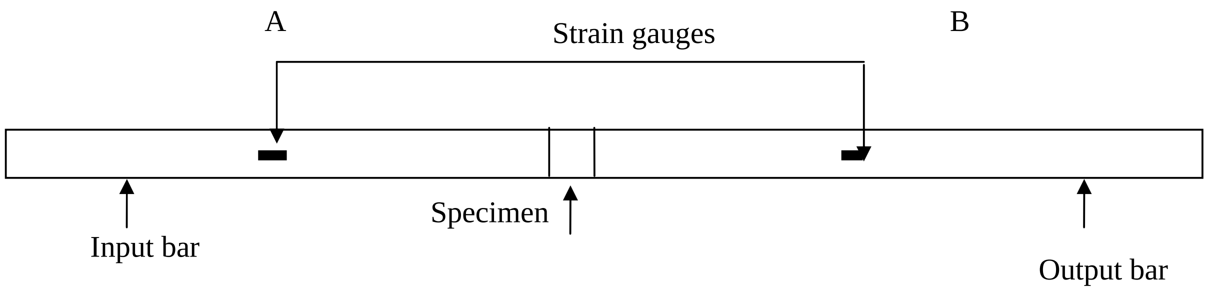
We now discuss the higher order terms in the asymptotic expansion of supplementary material.

The next term of the development is the T stress, which is uniform and induces also a uniform strain. Hence, the time derivative of strain for the strain is null.

Next terms of the stress and strain development are terms proportional to $r^{½+n}$, with *n* being an integer. The order of magnitude of the strain rate can be computed with $\frac{d\varepsilon}{dt} \cong \frac{d\varepsilon}{dr}\frac{dr}{dt} \cong v_{rupt}\frac{d\varepsilon}{dr}$, by remembering that $r = \sqrt{(x - vt)^2 + y_0^2}$. Therefore, if a strain scales as $\varepsilon = r^n$, its time derivative scales as scales as $\dot{\varepsilon} = v_{rupt} \times r^{n-1} = \frac{v_{rupt}}{r}\varepsilon$. If we limit the strain to 2%, a rupture velocity of 5000m/s and a distance to the fault equal to y_0=100m, the maximum strain rate is of the order of 1/s, about 100 times smaller than the experimental threshold for the transition to single fracturing to pulverization.

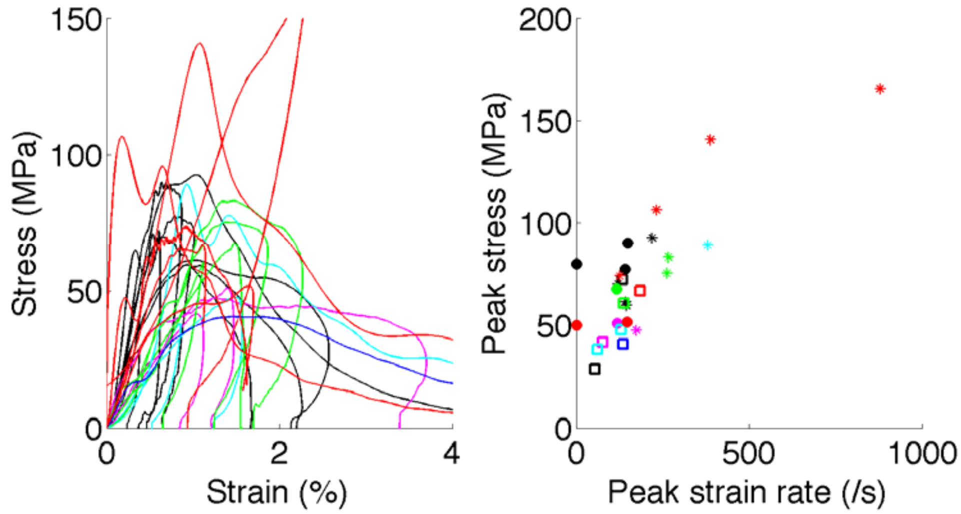


Supplementary Figure 1. Summary of the main findings of this paper. We show that rocks from the San Andreas Fault are pulverized if they are loaded at a rate larger than 150/s. Such high strain rate are not easy to obtain with a subshear earthquake, but may be reached in the Mach cone of a supershear earthquake.

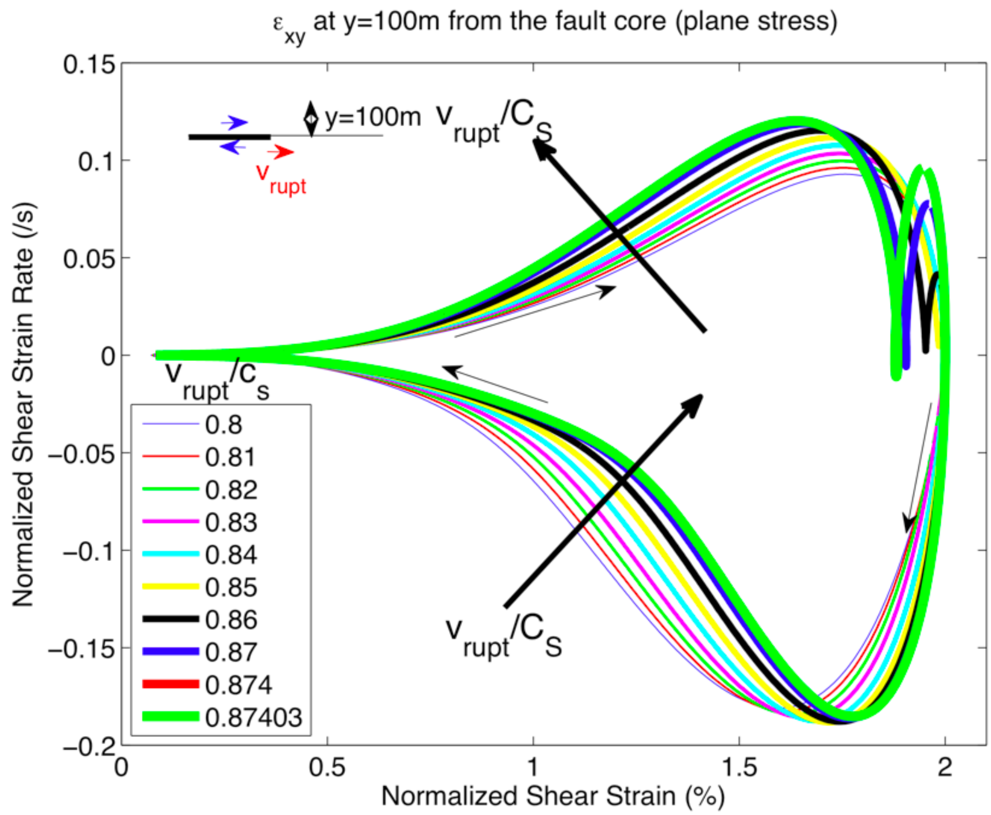


Supplementary figure 2. Sketch of the Split Hopkinson Pressure Bar apparatus. The sample is inserted between two long bars, before being submitted to the stress wave generated by the impact of the striker.

In the standard SHPB configuration (Kolsky) the strain measured in A shows a first wave induced by the loading (the incident wave) followed by its reflection at the specimen face (the reflected wave). The strain measured in B shows a wave induced by the loaded specimen (the transmitted wave). Subsequent waves are not recorded as they are superposed at gauge locations and cannot be easily identified. A very precise measurement of time (with modern data acquisition systems) and a very precise 3-D modeling of the wave propagation in bars (based on the Pochhammer and Chree equations) allows for the computation of forces and displacements at both specimen ends. When the quasi-equilibrium of the specimen is checked (equality of input and output forces) stress, stress and strain-rate are derived. A direct measurement of the Young’s modulus of the specimen is also possible, based on the transient 1-D analysis of the test.



Supplementary figure 3. Summary of the 27 experiments performed. Each sample was cored from a block. We grouped the samples by block, with one colour for each block. On the left, we show the obtained stress-strain curves. On the right, we reproduce the results of figure 2. We used the same conventions as in Figure 2 to denote the results of an experiment: a star denotes a pulverized sample, a filled circle a split sample and an empty square an unbroken sample. Some blocks (red and black) are tougher than the others, but the transition lies within the same region of strain rate for all blocks.



Supplementary figure 4. Shear strain rate $d\varepsilon_{xy}$ /dt versus shear strain ε_{xy} induced by a rupture propagating with a constant below the Rayleigh velocity in a location 100 m away from the fault core. As the fracture tip passes near the monitoring location strain and strain rate varies. We normalized both the strain and strain rate so that the maximum strain amplitude is 2%, corresponding to the order of magnitude of the maximum strain experienced by the pulverized rocks near the San Andreas Fault. The curves were computed for several rupture velocities. All curves achieve a maximum strain rate less than 0.25/s, three orders of magnitude below the 100/s pulverization threshold we determined in the laboratory. Details of the computation are given in early parts of the supplementary material.